Cambridge International AS & A Level

MATHEMATICS (9709) P3

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS

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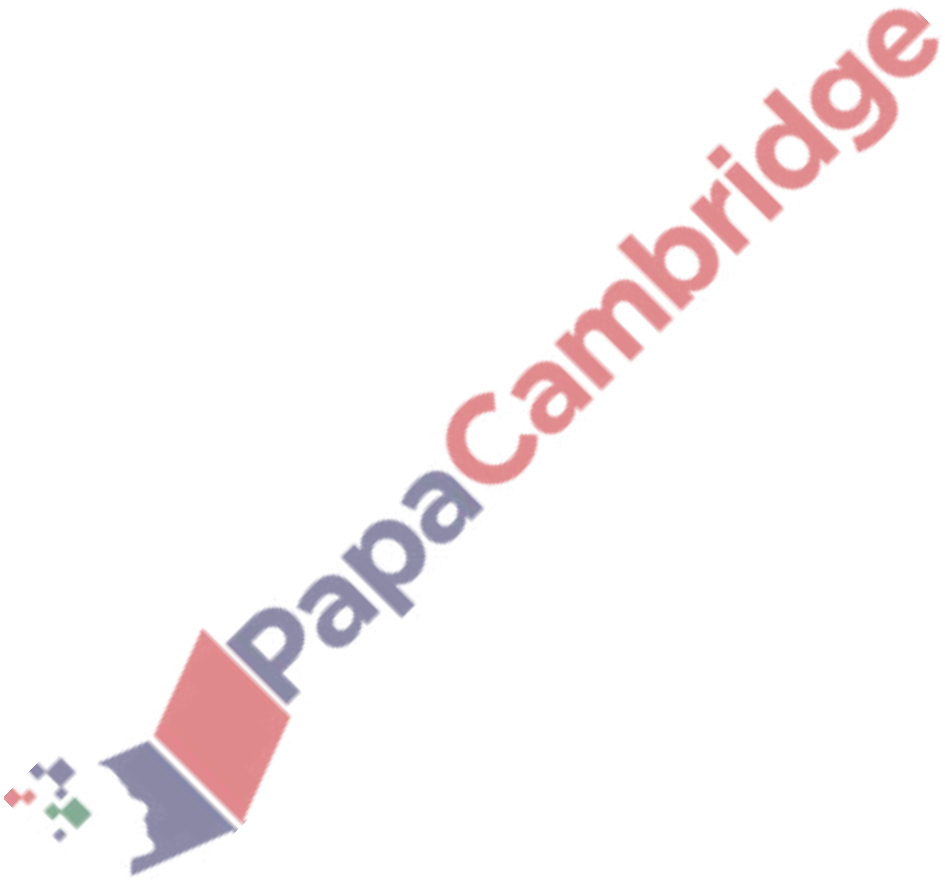
Chapter 2

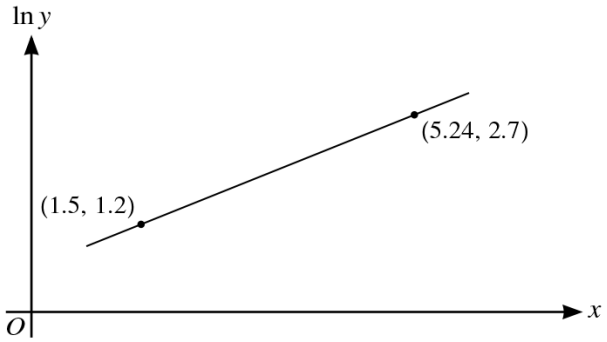
Logarithmic and exponential functions



47. 9709_s20_qp_31 Q: 1

Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

48. 9709_s20_qp_32 Q: 2



The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points (1.5, 1.2) and (5.24, 2.7) as shown in the diagram.

Find the values of A and k correct to 2 decimal places. [5]

49. 9709_s20_qp_33 Q: 3

(a) Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x. [2]

(b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places. [4]

50. 9709_w20_qp_31 Q: 4

Solve the equation

$$\log_{10}(2x + 1) = 2\log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

51. 9709_w20_qp_32 Q: 1

Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places. [3]

52. 9709_w20_qp_32 Q: 3

The variables x and y satisfy the relation $2^y = 3^{1-2x}$.

(a) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

(b) Find the exact x-coordinate of the point of intersection of this line with the line $y = 3x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [2]

53. 9709_m19_qp_32 Q: 1

(i) Show that the equation $\log_{10}(x-4) = 2 - \log_{10} x$ can be written as a quadratic equation in x. [3]

(ii) Hence solve the equation $\log_{10}(x-4) = 2 - \log_{10} x$, giving your answer correct to 3 significant figures. [2]

54. 9709_s19_qp_31 Q: 2

Showing all necessary working, solve the equation $\ln(2x - 3) = 2\ln x - \ln(x - 1)$. Give your answer correct to 2 decimal places. [4]

55. 9709_s19_qp_32 Q: 2

Showing all necessary working, solve the equation $9^x = 3^x + 12$. Give your answer correct to 2 decimal places. [4]

56. 9709_s19_qp_33 Q: 1

Use logarithms to solve the equation $5^{3-2x} = 4(7^x)$, giving your answer correct to 3 decimal places. [4]

57. 9709_w19_qp_31 Q: 1

Given that $\ln(1+e^{2y}) = x$, express y in terms of x. [3]

58. 9709_w19_qp_32 Q: 1

Solve the equation $5\ln(4 - 3^x) = 6$. Show all necessary working and give the answer correct to 3 decimal places. [3]

59. 9709_w19_qp_33 Q: 3

Showing all necessary working, solve the equation $\frac{3^{2x}+3^{-x}}{3^{2x}-3^{-x}} = 4$. Give your answer correct to 3 decimal places. [4]

60. 9709_m18_qp_32 Q: 4

The variables x and y satisfy the equation $y^n = Ax^3$, where n and A are constants. It is given that $y = 2.58$ when $x = 1.20$, and $y = 9.49$ when $x = 2.51$.

(i) Explain why the graph of $\ln y$ against $\ln x$ is a straight line. [2]

(ii) Find the values of n and A, giving your answers correct to 2 decimal places. [4]

61. 9709_s18_qp_31 Q: 1

Showing all necessary working, solve the equation $\ln(x^4 - 4) = 4 \ln x - \ln 4$, giving your answer correct to 2 decimal places. [4]

62. 9709_s18_qp_32 Q: 1

Showing all necessary working, solve the equation $3|2^x - 1| = 2^x$, giving your answers correct to 3 significant figures. [4]

63. 9709_s18_qp_33 Q: 2

Showing all necessary working, solve the equation $5^{2x} = 5^x + 5$. Give your answer correct to 3 decimal places. [5]

64. 9709_w18_qp_31 Q: 2

Showing all necessary working, solve the equation $\frac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

65. 9709_w18_qp_32 Q: 4

Showing all necessary working, solve the equation

$$\frac{e^x + e^{-x}}{e^x + 1} = 4,$$

giving your answer correct to 3 decimal places. [5]

66. 9709_m17_qp_32 Q: 1

Solve the equation $\ln(1 + 2^x) = 2$, giving your answer correct to 3 decimal places. [3]

67. 9709_s17_qp_32 Q: 1

Solve the equation $\ln(x^2 + 1) = 1 + 2\ln x$, giving your answer correct to 3 significant figures. [3]

68. 9709_s17_qp_33 Q: 3

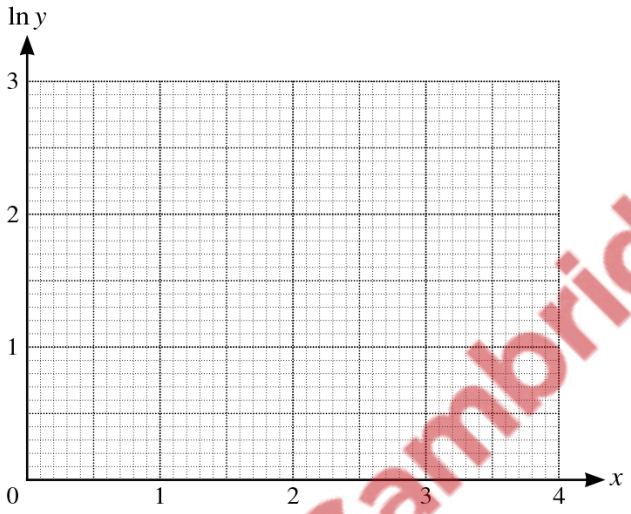
Using the substitution $u = e^x$, solve the equation $4e^{-x} = 3e^x + 4$. Give your answer correct to 3 significant figures. [4]

69. 9709_w17_qp_31 Q: 2

Two variable quantities x and y are believed to satisfy an equation of the form $y = C(a^x)$, where C and a are constants. An experiment produced four pairs of values of x and y. The table below gives the corresponding values of x and $\ln y$.

x	0.9	1.6	2.4	3.2
$\ln y$	1.7	1.9	2.3	2.6

By plotting $\ln y$ against x for these four pairs of values and drawing a suitable straight line, estimate the values of C and a. Give your answers correct to 2 significant figures. [5]



70. 9709_w17_qp_32 Q: 2

Showing all necessary working, solve the equation $2\log_2 x = 3 + \log_2(x+1)$, giving your answer correct to 3 significant figures. [5]

71. 9709_m16_qp_32 Q: 1

Solve the equation $\ln(x^2 + 4) = 2\ln x + \ln 4$, giving your answer in an exact form. [3]

72. 9709_s16_qp_32 Q: 1

Use logarithms to solve the equation $4^{3x-1} = 3(5^x)$, giving your answer correct to 3 decimal places. [4]

73. 9709_s16_qp_33 Q: 2

The variables x and y satisfy the relation $3^y = 4^{2-x}$.

(i) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

(ii) Calculate the exact x-coordinate of the point of intersection of this line with the line with equation $y = 2x$, simplifying your answer. [2]

74. 9709_w16_qp_31 Q: 1

Solve the equation $\frac{3^x + 2}{3^x - 2} = 8$, giving your answer correct to 3 decimal places. [3]

75. 9709_w16_qp_33 Q: 1

It is given that $z = \ln(y + 2) - \ln(y + 1)$. Express y in terms of z. [3]

76. 9709_s15_qp_31 Q: 1

Use logarithms to solve the equation $2^{5x} = 3^{2x+1}$, giving the answer correct to 3 significant figures. [4]

77. 9709_s15_qp_32 Q: 2

Using the substitution $u = 4^x$, solve the equation $4^x + 4^2 = 4^{x+2}$, giving your answer correct to 3 significant figures. [4]

78. 9709_s15_qp_33 Q: 1

Solve the equation $\ln(x+4) = 2\ln x + \ln 4$, giving your answer correct to 3 significant figures. [4]

79. 9709_w15_qp_31 Q: 2

Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures. [5]

80. 9709_w15_qp_33 Q: 1

Sketch the graph of $y = e^{ax} - 1$ where a is a positive constant. [2]